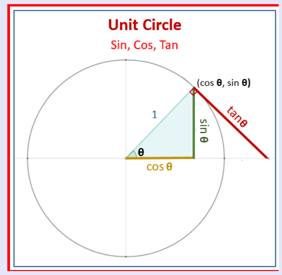
Trigonometry

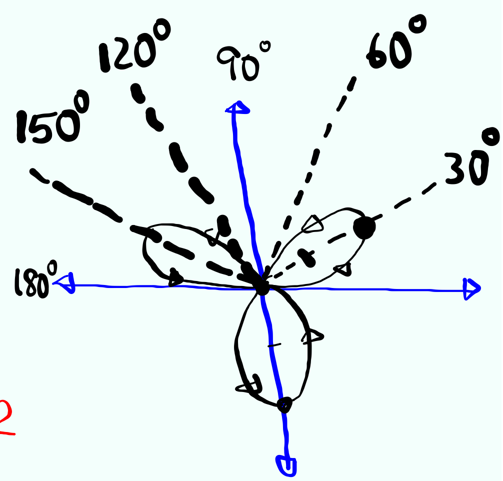
Lecture 51



Feb 19-8:47 AM

Graph $r = 2\sin 3\theta$

$3\theta = 0^\circ$	$\theta = 0^\circ$	$r = 0$
$3\theta = 90^\circ$	$\theta = 30^\circ$	$r = 2$
$3\theta = 180^\circ$	$\theta = 60^\circ$	$r = 0$
$3\theta = 270^\circ$	$\theta = 90^\circ$	$r = -2$
$3\theta = 360^\circ$	$\theta = 120^\circ$	$r = 0$
$3\theta = 450^\circ$	$\theta = 150^\circ$	$r = 2$
$3\theta = 540^\circ$	$\theta = 180^\circ$	$r = 0$



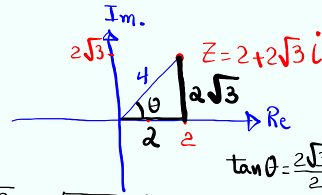
$r = 2\sin 3\theta$

Rose

3 petals

Dec 4-10:27 AM

$Z = 2 + 2\sqrt{3}\,i$



$Re. = 2$

$Im. = 2\sqrt{3}$

$\tan\theta = \frac{2\sqrt{3}}{2}$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

$|Z| = \sqrt{Re^2 + Im^2} = \sqrt{2^2 + (2\sqrt{3})^2}$

Abs. value of Z

Modulus of Z

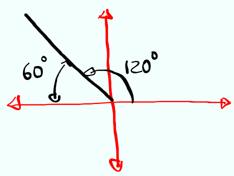
$= \sqrt{4 + 12} = \sqrt{16} = \boxed{4}$

$Z = 2 + 2\sqrt{3}\,i = 4(\cos 60^\circ + i \sin 60^\circ)$

Complex Form — Polar or Trig form

$Z^2 = 4^2(\cos 2\cdot 60^\circ + i \sin 2\cdot 60^\circ)$

$= 16(\cos 120^\circ + i \sin 120^\circ)$



$= 16(-\cos 60^\circ + i \sin 60^\circ)$

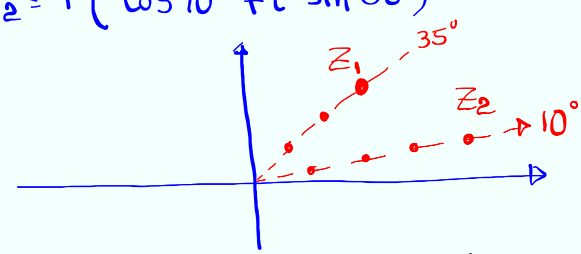
$= 16\left(-\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right)$

$= \boxed{-8 + 8\sqrt{3}\,i}\ Z^2$

Dec 4-10:35 AM

$Z_1 = 3(\cos 35^\circ + i \sin 35^\circ)$

$Z_2 = 4(\cos 10^\circ + i \sin 30^\circ)$



Find $Z_1 Z_2 = 3\cdot 4(\cos(35^\circ + 10^\circ) + i \sin(35^\circ + 10^\circ))$

$= 12(\cos 45^\circ + i \sin 45^\circ)$

$\frac{Z_1}{Z_2} = \frac{3}{4}(\cos(35^\circ - 10^\circ) + i \sin(35^\circ - 10^\circ))$

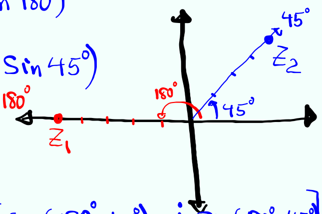
$= \frac{3}{4}(\cos 25^\circ + i \sin 25^\circ)$

Dec 4-10:43 AM

$Z_1 = 5(\cos 180^\circ + i\sin 180^\circ)$

$Z_2 = 4(\cos 45^\circ + i\sin 45^\circ)$

1) Plot Z_1 & Z_2



2) Find $Z_1 Z_2 = 5\cdot 4[\cos(180^\circ+45^\circ) + i\sin(180^\circ+45^\circ)]$

$= 20[\cos 225^\circ + i\sin 225^\circ] = \boxed{20\,\text{cis}\,225^\circ}$

3) Find $\frac{Z_1}{Z_2} = \frac{5}{4}[\cos(180^\circ-45^\circ) + i\sin(180^\circ-45^\circ)]$

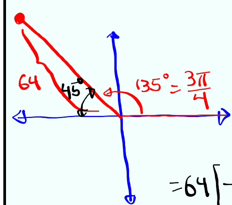
$= \frac{5}{4}[\cos 135^\circ + i\sin 135^\circ] = \frac{5}{4}\,\text{cis}\,135^\circ$

4) Find $Z_2^3 = 4^3[\cos 3\cdot 45^\circ + i\sin 3\cdot 45^\circ]$

$= 64[\cos 135^\circ + i\sin 135^\circ]$

$= 64\,\text{cis}\,135^\circ$

$= 64\,\text{cis}\,\frac{3\pi}{4}$



$= 64[-\cos 45^\circ + i\sin 45^\circ] = 64[-\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}]$

$= \boxed{-32\sqrt{2} + 32\sqrt{2}\,i}$

Dec 4-10:49 AM

If $Z = r(\cos\theta + i\sin\theta)$, for any positive integer n,

$$\sqrt[n]{Z} = \sqrt[n]{r}\left[\cos\frac{\theta + K\cdot 360^\circ}{n} + i\sin\frac{\theta + K\cdot 360^\circ}{n}\right]$$

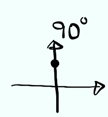
($\theta + K\cdot 2\pi$; $\theta + K\cdot 2\pi$)

for $K = 0, 1, 2, \ldots, n-1$.

$n=2$

ex: $Z^2 - 4i = 0$ $\qquad Z^2 = 4i$

$Z^2 = 4(0 + i)$



$Z^2 = 4(\cos 90^\circ + i\sin 90^\circ)$

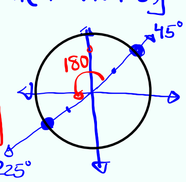
$Z = \sqrt{4}\left[\cos\frac{90^\circ + K\cdot 360^\circ}{2} + i\sin\frac{90^\circ + K\cdot 360^\circ}{2}\right]$

$Z = 2[\cos(45^\circ + \underline{K\cdot 180^\circ}) + i\sin(45^\circ + K\cdot 180^\circ)]$

$K=0$ $\qquad Z = 2[\cos 45^\circ + i\sin 45^\circ]$

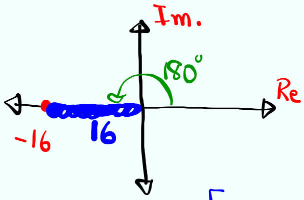
$K=1$ $\qquad Z = 2[\cos 225^\circ + i\sin 225^\circ]$

$n-1$



Dec 4-11:02 AM

$$Z^4 + 16 = 0 \qquad Z^4 = -16$$



$$Z^4 = -16 + 0i$$

$$Z^4 = 16\left(\cos 180° + i \sin 180°\right)$$

$$Z = \sqrt[4]{16}\left[\cos \frac{180° + K\cdot 360°}{4} + i \sin \frac{180° + K\cdot 360°}{4}\right]$$

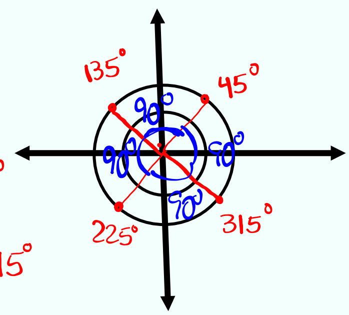
$$= 2\left[\cos(45° + K\cdot \underline{90°}) + i \sin(45° + K\cdot 90°)\right]$$

$K=0 \quad Z_1 = 2\,\text{CiS}\,45°$

$K=1 \quad Z_2 = 2\,\text{CiS}\,135°$

$K=2 \quad Z_3 = 2\,\text{CiS}\,225°$

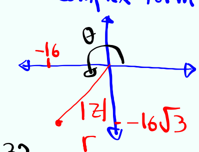
$K=3 \leftarrow n-1 \quad Z_4 = 2\,\text{CiS}\,315°$



Dec 4-11:12 AM

find the fifth roots of $-16 - 16\sqrt{3}\,i$.

$\underbrace{n=5}_{\text{Five Answers.}}$ Complex Form



$$r = |z| = \sqrt{(-16)^2 + (-16\sqrt{3})^2}$$

$$= \sqrt{256 + 256\cdot 3} = \sqrt{1024} = 32$$

$\tan\theta = \frac{16\sqrt{3}}{16}$

$\tan\theta = \sqrt{3}$

$RA = 60°$

$$-16 - 16\sqrt{3}\,i = 32\left[\cos 240° + i \sin 240°\right]$$

$\theta = 180° + 60° = 240°$

$$Z = \sqrt[5]{32}\left[\cos \frac{240° + K\cdot 360°}{5} + i \sin \frac{240° + K\cdot 360°}{5}\right]$$

$$Z = 2\left[\cos(48° + K\cdot 72°) + i \sin(48° + K\cdot 72°)\right]$$

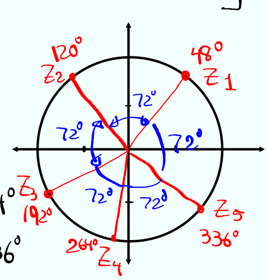
$K=0 \rightarrow Z_1 = 2\,\text{CiS}\,48°$

$K=1 \rightarrow Z_2 = 2\,\text{CiS}\,120°$

$K=2 \rightarrow Z_3 = 2\,\text{CiS}\,192°$

$K=3 \rightarrow Z_4 = 2\,\text{CiS}\,264°$

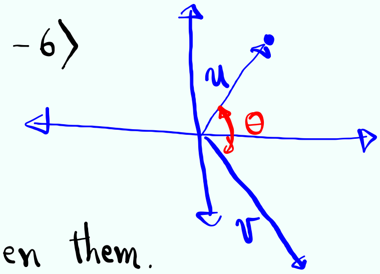
$K=4 \rightarrow Z_5 = 2\,\text{CiS}\,336°$



Dec 4-11:21 AM

$u = \langle 3, 5 \rangle$ $\quad v = \langle 2, -6 \rangle$

1) Draw u & v



2) Find the angle between them.

$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$u \cdot v = 3(2) + 5(-6) = 6 - 30 = -24$$

$$|u| = \sqrt{3^2 + 5^2} = \sqrt{34}$$

$$\cos\theta = \frac{-24}{\sqrt{34}\sqrt{40}}$$

$$|v| = \sqrt{2^2 + (-6)^2} = \sqrt{40}$$

$$\cos\theta = -.651 \qquad \theta = \cos^{-1}(-.651)$$

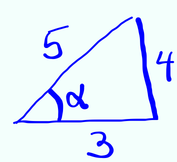
$$\boxed{\theta \approx 131^\circ}$$

Dec 4-11:36 AM

find exact value of $\sin\left(2\underbrace{\cos^{-1}\frac{3}{5}}_{\alpha}\right)$

$$\alpha = \cos^{-1}\frac{3}{5}$$

$$\cos\alpha = \frac{3}{5}$$



$$\sin 2\alpha$$

$$= 2\sin\alpha\cos\alpha$$

$$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \boxed{\frac{24}{25}}$$

Dec 4-11:41 AM